Name...**Stream**.........**House**.......................

DEPARTMENT OF MATHEMATICS

S.4 MATHEMATICS−2020

REVISION QUESTIONS

VECTORS

1. Points A and B have coordinates $(0, -1)$ and $(-6, 7)$ respectively. Find:

   (a) $\mathbf{AB}$,

   (b) magnitude of $\mathbf{AB}$. *(04 marks)*

2. Two points $P(5, 2)$ and $Q(2, 4)$ are in a plane. Find the

   (a) coordinate of M, the midpoint of $\mathbf{PQ}$,

   (b) $|\mathbf{OM}|$, where O is the origin. *(04 marks)*

3. If $\mathbf{a} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 10 \\ 9 \end{pmatrix}$, find:

   (i) $\frac{1}{2}(\mathbf{a} - \mathbf{c})$,

   (ii) $|\mathbf{a} + \mathbf{b} - \mathbf{c}|$. *(05 marks)*

4. P and Q have position vectors $\begin{pmatrix} 2t \\ t+1 \end{pmatrix}$, $\begin{pmatrix} t+1 \\ -(t+2) \end{pmatrix}$ respectively. If $|\mathbf{OP}| = |\mathbf{OQ}|$, show that $3t^2 - 4t - 4 = 0$ and hence find the possible values of t. *(05 marks)*

5. In a parallelogram $OBCA$, $\mathbf{OA} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{OB} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, where O is the origin. Find

   (i) $\mathbf{BC}$,

   (ii) the coordinates of C. *(04 marks)*

6. Given that $\begin{pmatrix} a \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$, find values of a and b. *(04 marks)*

7. Given vectors $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 18 \\ 3 \end{pmatrix}$, find the values of the constants p and q such that $\mathbf{c} = p\mathbf{a} + q\mathbf{b}$. *(04 marks)*

8. If $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$, find numbers m and n such that $m\mathbf{a} + n\mathbf{b} = \mathbf{c}$. *(04 marks)*

9. Given that $S(-2, 6)$ and $T(3, 3)$ are two points, find the coordinates of R if $\mathbf{OR} = 4\mathbf{OS} + \frac{1}{3}\mathbf{OT}$ and O is the origin. *(04 marks)*

10. Given that $\mathbf{OA} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, $\mathbf{OB} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ and M is a point on AB such that $AM : MB = 1 : 1$; find:

    (a) $\mathbf{AM}$.

    (b) $\mathbf{OM}$. *(04 marks)*

11. (a) Given the vectors $\mathbf{a} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, find:

    (i) $\mathbf{a} + 2\mathbf{b} + \mathbf{c}$.

    (ii) length of $\mathbf{a} + 2\mathbf{b} + \mathbf{c}$.

    (b) The position vectors of D and E are $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ -11 \end{pmatrix}$ respectively. M is on DE such that $DM : DE = 2 : 3$. Find

    (i) $\mathbf{DE}$,

    (ii) $\mathbf{DM}$,

    (iii) the position vector of M.

12. Given that $\mathbf{p} = 3\mathbf{a} - \mathbf{b}$ and $\mathbf{q} = 2\mathbf{a} - 3\mathbf{b}$, find the numbers x and y such that $x\mathbf{p} + y\mathbf{q} = \mathbf{a} + 9\mathbf{b}$. *(04 marks)*

13. Given that $\mathbf{p} = 2\mathbf{a} - 5\mathbf{b}$, $\mathbf{q} = \mathbf{a} + 2\mathbf{b}$ and $\mathbf{r} = \mathbf{a} - 16\mathbf{b}$, find the numbers m and n such that $m\mathbf{p} + n\mathbf{q} = \mathbf{r}$. *(04 marks)*

14. Show that points $A(1,2)$, $B(2,4)$ and $C(4,8)$ are collinear. *(04 marks)*

15. Show that points $A(1,1)$, $B(2,3)$ and $C(5,9)$ lie on a straight line. *(04 marks)*

16. If points $P(1,1)$, $Q(2,4)$ and $R(p,-1)$ are collinear find the value of p. *(04 marks)*

17. If $\mathbf{OP} = 2\mathbf{a} - 5\mathbf{b}$, $\mathbf{OQ} = 5\mathbf{a} - \mathbf{b}$ and $\mathbf{OR} = 11\mathbf{a} + 7\mathbf{b}$, show that P, Q and R are collinear and state the ratio $PQ : QR$. *(04 marks)*

18. The position vectors of P, Q and R are $\mathbf{a} - 2\mathbf{b}$, $2\mathbf{b}$ and $-4\mathbf{a} + k\mathbf{b}$ respectively. If P, Q and R are collinear, find the value of k. What is the ratio $PQ : QR$? *(04 marks)*

19. If $\mathbf{OP} = \mathbf{a} + \mathbf{b}$, $\mathbf{OQ} = k\mathbf{a}$ and $\mathbf{OR} = 7\mathbf{a} - 2\mathbf{b}$, find the value of k if Q lies on PR. *(04 marks)*

20. The position vectors of P, Q and R are $2\mathbf{a} - \mathbf{b}$, $t(\mathbf{a} - \mathbf{b})$ and $\mathbf{a} + \mathbf{b}$ respectively. Find the value of t if PQR is a straight line. State the ratio $PQ : QR$. *(04 marks)*

21. Show that the points A, B and C with position vectors $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ respectively are collinear. *(04 marks)*

22. If the vectors $\mathbf{a} = \begin{pmatrix} m \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ are parallel, find the value of m. *(04 marks)*

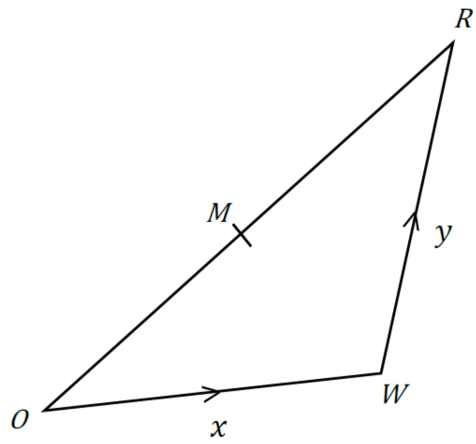
23. The position vectors of A and B are $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} t \\ 1 \end{pmatrix}$ respectively. Find the value of t if OAB is a straight line, where O is the origin. *(04 marks)*

24. P and Q divide the sides BC and AC respectively of $\triangle ABC$ in the ratio $2 : 1$. If $\mathbf{AB} = \mathbf{a}$ and $\mathbf{AC} = \mathbf{b}$, find

    (a) $\mathbf{QP}$ and

    (b) Show that $\mathbf{QP}$ is parallel to $\mathbf{AB}$. *(08 marks)*

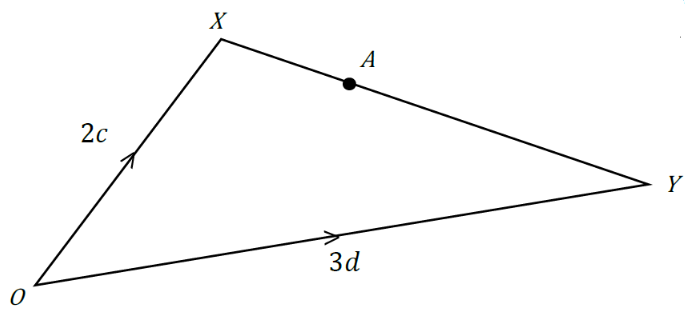
25. In the traingle ORW below, $\mathbf{OW} = \mathbf{x}$, $\mathbf{WR} = \mathbf{y}$. M is the midpoint of OR.



Find interms of **x** and **y**

(a) **OR**. (b) **OM**. (c) **WM**. (*06 marks*)

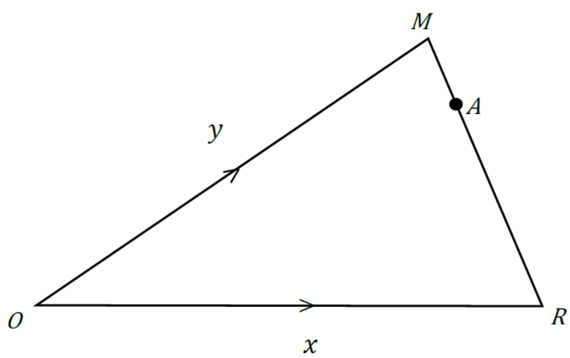
26. In the traingle OXY below, $\mathbf{OX} = 2\mathbf{c}$, $\mathbf{OY} = 3\mathbf{d}$. A is the point on XY such that $XA : AY = 2 : 3$



(a) Find **XY** interms of **c** and **d**

(b) Show that **OA** is parallel to the vector $\mathbf{c} + \mathbf{d}$. (*04 marks*)

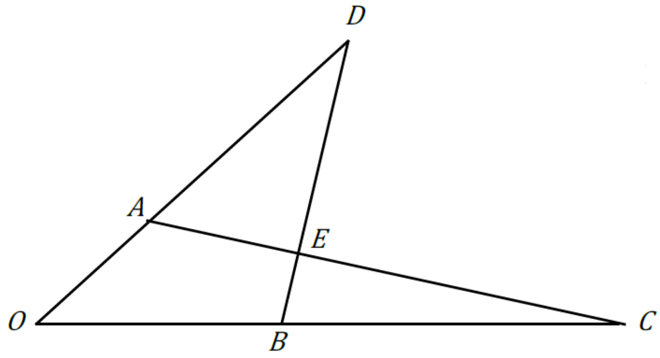
27. In the traingle OMR below, $\mathbf{OR} = \mathbf{x}$, $\mathbf{OM} = \mathbf{y}$. A is the point on MR such that $MA : AR = 3 : 1$



Find interms of **x** and **y**

(a) **RM**. (b) **OA**. (*06 marks*)

28. In the below, $\mathbf{OA} = 4\mathbf{x}$, $\mathbf{OB} = 4\mathbf{y}$. OAD, OBC and BED are all straight lines. $AD = 2OA$ and $BE : ED = 1 : 3$.



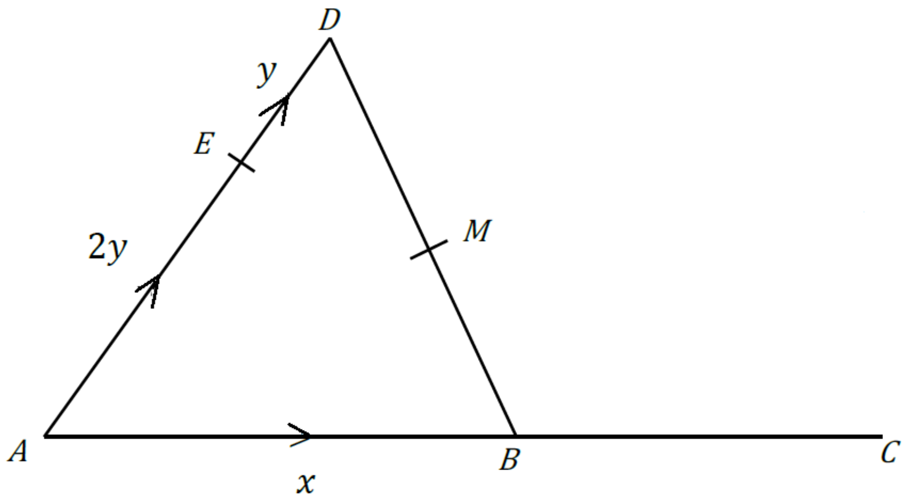
(a) Find, in terms of $\mathbf{x}$ and $\mathbf{y}$, the vectors which represent:

(i) $\mathbf{BD}$. (ii) $\mathbf{AE}$.

(b) Given that $\mathbf{BC} = 8\mathbf{y}$. Show that points A, E and C lie on a straight line.

(*06 marks*)

29. In the triangle ABD, $\mathbf{AB} = \mathbf{x}$, $\mathbf{AE} = 2\mathbf{y}$ and $\mathbf{ED} = \mathbf{y}$. E is a point on AD, B is the midpoint of AC and M is the midpoint of DB.

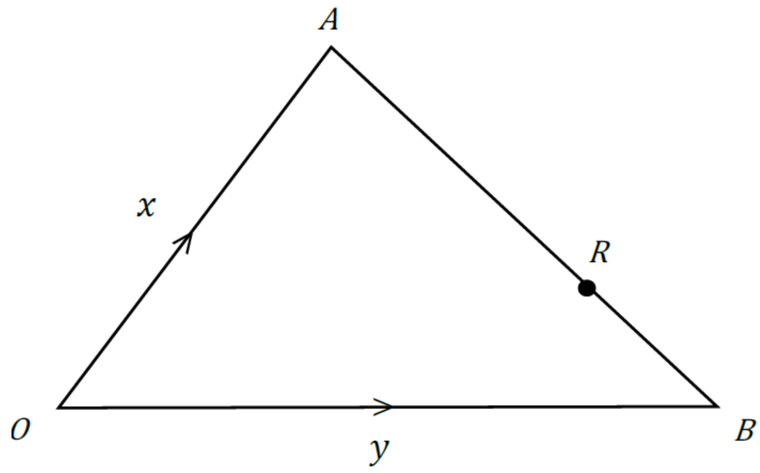


(a) Find, in terms of $\mathbf{x}$ and $\mathbf{y}$, the vectors which represent:

(i) $\mathbf{DA}$. (ii) $\mathbf{DB}$. (iii) $\mathbf{AM}$.

(b) Show that points E, M and C lie on a straight line. (*08 marks*)

30. In the traingle OAB below, $\mathbf{OA} = \mathbf{x}$, $\mathbf{OB} = \mathbf{y}$. R is the point on AB such that $AR : RB = 3 : 2$

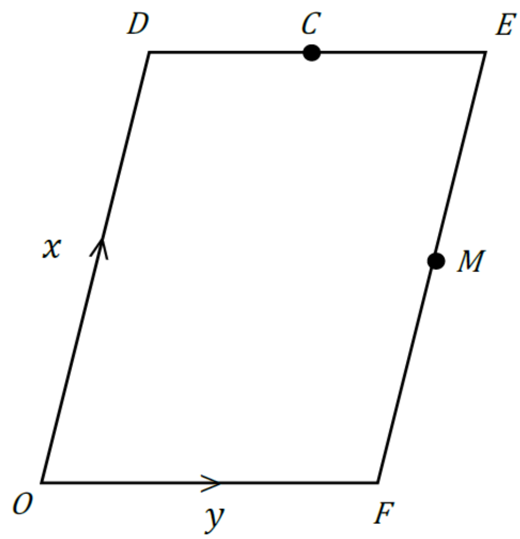


Find interms of $\mathbf{x}$ and $\mathbf{y}$

(a) $\mathbf{AB}$.

(b) Show that $\mathbf{OR} = \frac{1}{5}(2\mathbf{x} + 3\mathbf{y})$. *(06 marks)*

31. In the parallelogram $ODEF$ below, $\mathbf{OD} = \mathbf{x}$, $\mathbf{OF} = \mathbf{y}$. M is the midpoint of FE. C is the midpoint of DE.

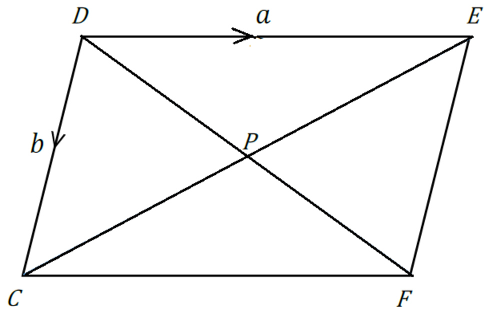


(a) Express, in terms of $\mathbf{x}$ and/or $\mathbf{y}$, the vectors:

(i) $\mathbf{ME}$. (ii) $\mathbf{MC}$.

(b) Show that $\mathbf{FD}$ is parallel to $\mathbf{MC}$. *(06 marks)*

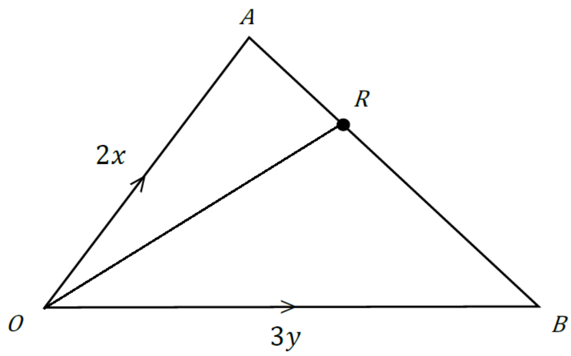
32. In the parallelogram $CDEF$ below, $\mathbf{DE} = \mathbf{a}$, $\mathbf{DC} = \mathbf{b}$. DF and EC are diagonals of parallelogram $DEFC$ and they intersect at point P.



Express, in terms of **a** and **b**, the vectors:

(a) **DF**. (b) **EC**. (c) **DP**. (*06 marks*)

33. In the traingle OAB below, $\mathbf{OA} = 2\mathbf{x}$, $\mathbf{OB} = 3\mathbf{y}$. R is the point on AB such that $AR : RB = 2 : 3$

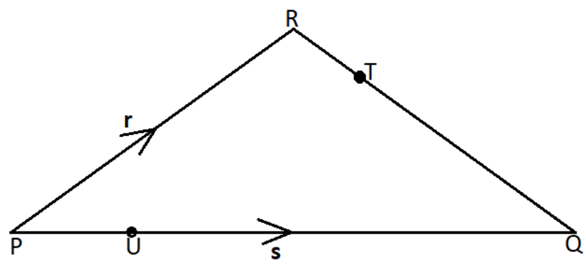


Find interms of **x** and **y**

(a) **AB**.

(b) Show that **OR** is parallel to the vector $\mathbf{x} + \mathbf{y}$. (*07 marks*)

34. In the figure below, vector, $\mathbf{PQ} = \mathbf{s}$, $\mathbf{PR} = \mathbf{r}$, $2QT = 3TR$ and $PU : UQ = 2 : 3$.

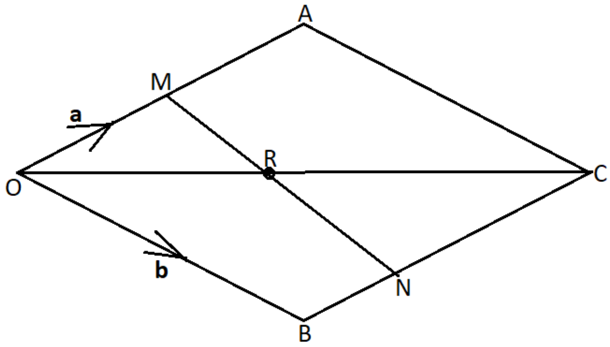


(a) Find in terms of **r** and **s**, vectors

(i) **QR**, (ii) **QT**, (iii) **PT**

(b) Show that **PT** is parallel to **PR**. (*12 marks*)

35. The diagram below shows a parallelogram $OACB$ .The position vectors of A and B are **a** and **b** respectively.

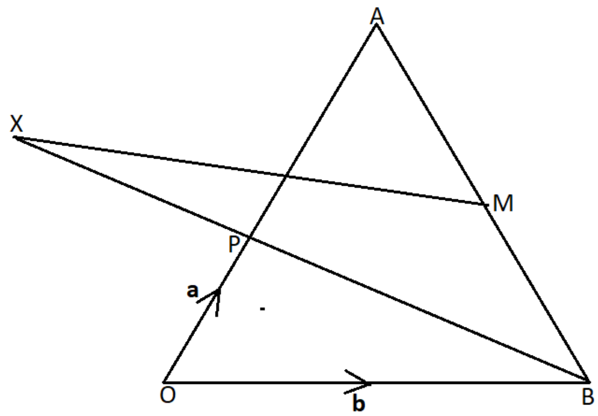


(a) If $OM = MA$ and $NC = 2BN$, find in terms of **a** and **b**

(i) **NC**. (ii) **NA**. (iii) **MN**.

(b) Given that $\mathbf{OR} = \lambda\mathbf{OC}$ and $\mathbf{MR} = \mu\mathbf{MN}$, find the value of λ and μ. Hence show that $\mathbf{AR} = \frac{1}{7}(3\mathbf{b} - 4\mathbf{a})$. (*12 marks*)

36. In the below, $\mathbf{OP} = \mathbf{a}$, $\mathbf{OA} = 3\mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. M is the midpoint of AB.



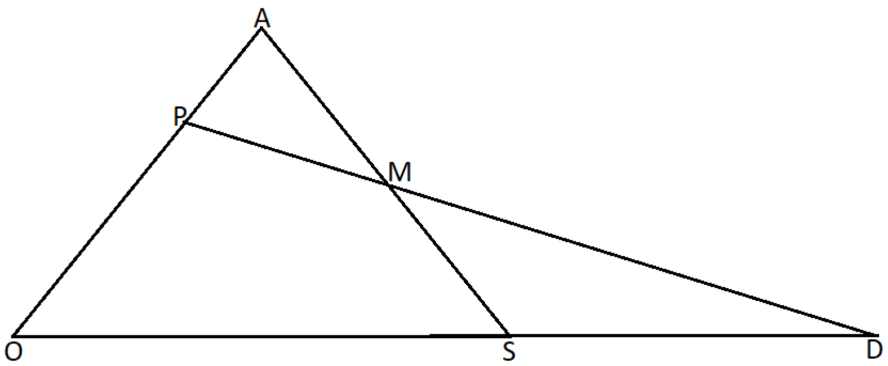
(a) Find, in terms of **a** and **b**

(i) **BP**. (ii) **AB**. (iii) **MB**.

(b) If X lies on BP produced such that $\mathbf{BX} = k\mathbf{BP}$, express **MX** in terms of **a**, **b** and k.

(c) Find the value of k if **MX** is parallel to **BO**. (*12 marks*)

37. In the below, $\mathbf{OA} = \mathbf{a}$, $\mathbf{OS} = \mathbf{b}$ and M is the midpoint of AB. The point P divides OA in the ratio $2 : 1$, and PM produced meets OS produced at D.



(a) Find, in terms of **a** and **b**

(i) **AS**. (ii) **PA**. (iii) **PM**.

(b) Given that $\mathbf{PD} = k\mathbf{PM}$ and $\mathbf{OD} = t\mathbf{OS}$, find the value of k and t.

(*12 marks*)

38. The position vectors of the points A and B are **a** and **b** respectively. The point P lies on OA produced such that $OP = 3OA$. Point Q lies on OB such that $OQ = \frac{1}{3}OB$. The lines AB and PQ meet at C

(a) (i) Express **PQ** in terms of vectors **a** and **b**.

(ii) If also $PC = \mu PQ$, express OC in terms of μ, **a** and **b**.

(iii) If also $AC = \lambda AB$, express OC in terms of λ, **a** and **b**. Hence find the value of λ and μ.

(b) Show that **AQ** is parallel to **PB**. (*12 marks*)

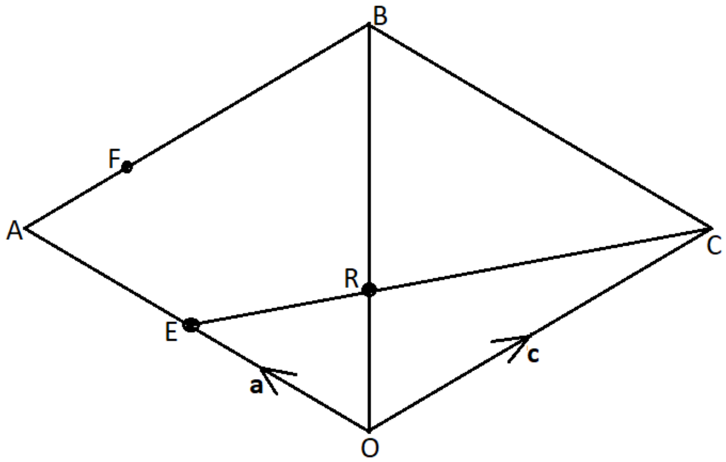
39. In a triangle ABC, M and D are mid−points of AC and CN respectively. N is a point on AB such that $AN = 3NB$.

(a) If $\mathbf{AB} = \mathbf{p}$ and $\mathbf{AC} = \mathbf{q}$, express the following vectors in terms of vectors **p** and **q**;

(i) **AM**. (ii) **AN**. (iii) **ND**.

(b) Show that **MD** is parallel to **AB** and that $MD : AB = 3 : 8$. (*12 marks*)

40. In the diagram below, $OABC$ is a parallelogram. $\mathbf{OA} = \mathbf{a}$, $\mathbf{OC} = \mathbf{c}$ and E is the midpoint of OA.



If $OR : RB = 1 : 2$ and $AF : FB = 1 : 5$,

(a) in terms of vectors $\mathbf{a}$ and $\mathbf{c}$ the vectors

(i) $\mathbf{FC}$. (ii) $\mathbf{FR}$.

(b) determine the ratio of $ER : RC$. (*12 marks*)

41. In a parallelogram $ABCD$, $\mathbf{AB} = 2\mathbf{a}$ and $\mathbf{AD} = \mathbf{b}$. A point E is such that $\mathbf{AE} = 2\mathbf{b}$. Lines AC and BD intersect at F. Lines CD and BE intersect at G. $\mathbf{DC} = 2\mathbf{DG}$. Find in terms of vectors $\mathbf{a}$ and $\mathbf{b}$

(a) $\mathbf{EC}$, (c) $\mathbf{BG}$,

(b) $\mathbf{AG}$, (d) $\mathbf{FG}$. (*12 marks*)

42. In a triangle ABC, E divides OA in the ratio $6 : 1$. D divides $\mathbf{AB}$ in the ratio $1 : 2$. Point C is on OB produced such that $OC : OB = 3 : 2$. Given that $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$,

(a) find in terms of vectors $\mathbf{a}$ and $\mathbf{b}$;

(i) $\mathbf{OD}$, (iii) $\mathbf{ED}$,

(ii) $\mathbf{AC}$, (iv) $\mathbf{DC}$.

(b) determine the ratio $ED : DC$. What can you conclude about the points E, D and C. (*12 marks*)

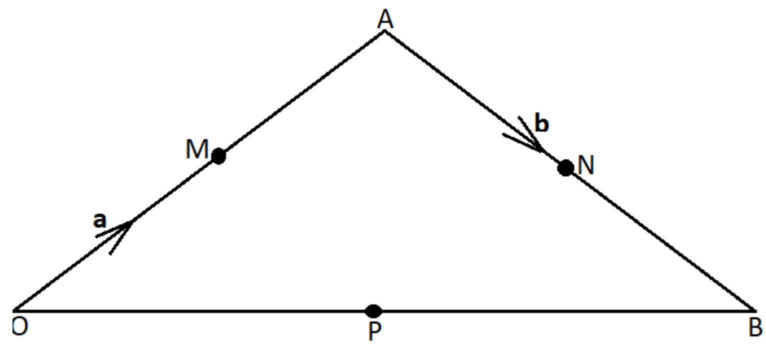
43. The points P and Q have position vectors $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R divides PQ externally in the ratio $3 : 1$. S is a point on a straight line with R such that $3RS = 2PQ$. Find the

(a) position vectors of R and S

(b) vectors $\mathbf{QS}$ and $\mathbf{PS}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(c) ratio $PS : RS$. *(12 marks)*

44. The vectors $\mathbf{OA}$, $\mathbf{AB}$ and $\mathbf{BO}$ form a triangle OAB with O the origin. If $\mathbf{OA} = \mathbf{a}$, $\mathbf{AB} = \mathbf{b}$, and the points M, N and P are midpoints of OA, AB and OB respectively.



(a) Determine in terms of vectors $\mathbf{a}$ and $\mathbf{b}$

(i) $\mathbf{AP}$, (ii) $\mathbf{NP}$, (iii) $\mathbf{MB}$.

(b) Show that $\mathbf{MN}$ is parallel to $\mathbf{OB}$.

(c) determine the ratio of $MN : OB$. *(12 marks)*

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